Quantum Electronic Phases in Partially Filled Landau Levels

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ECRYS 2005, Cargese August 22-27, 2005

Collaborators

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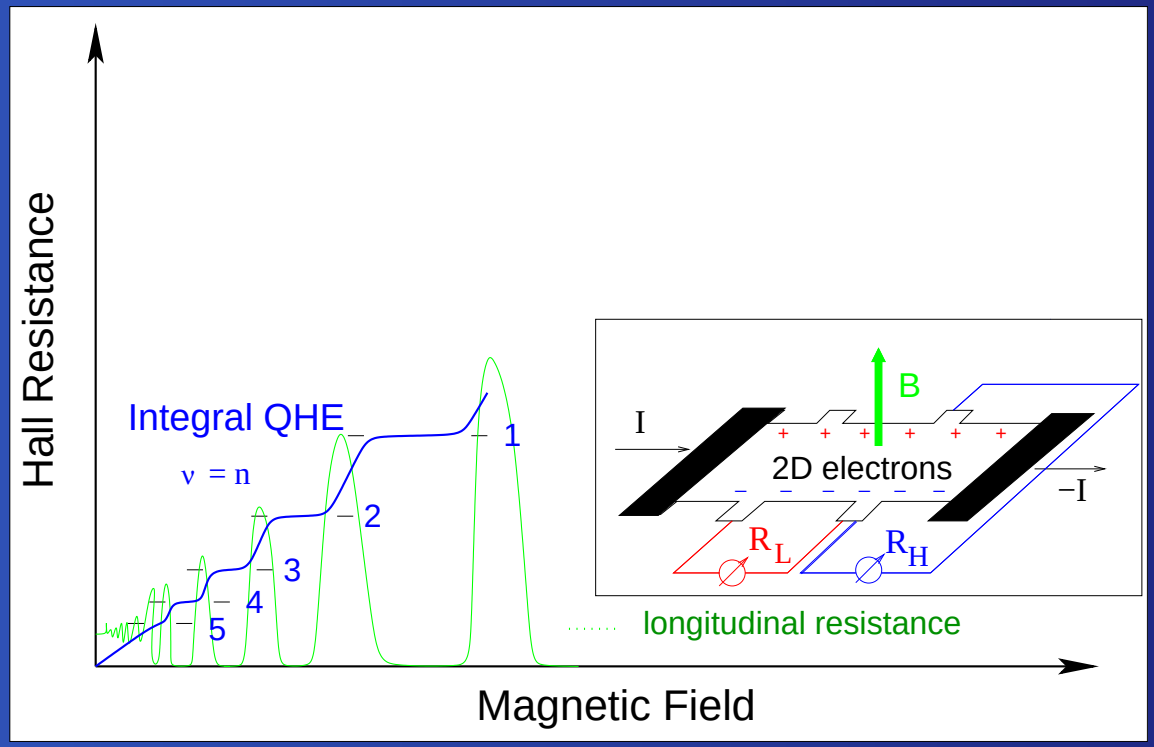
  Laboratoire de Physique des Solides, Université Paris-Sud (France).
- Cristiane Morais Smith
  Institute for Theoretical Physics, University of Utrecht (The Netherlands).

Outline

- Experiments
- Model and Energies of the Different Phases
- Results
- Phase Transitions

Framework: Quantum Hall Effects, 2D



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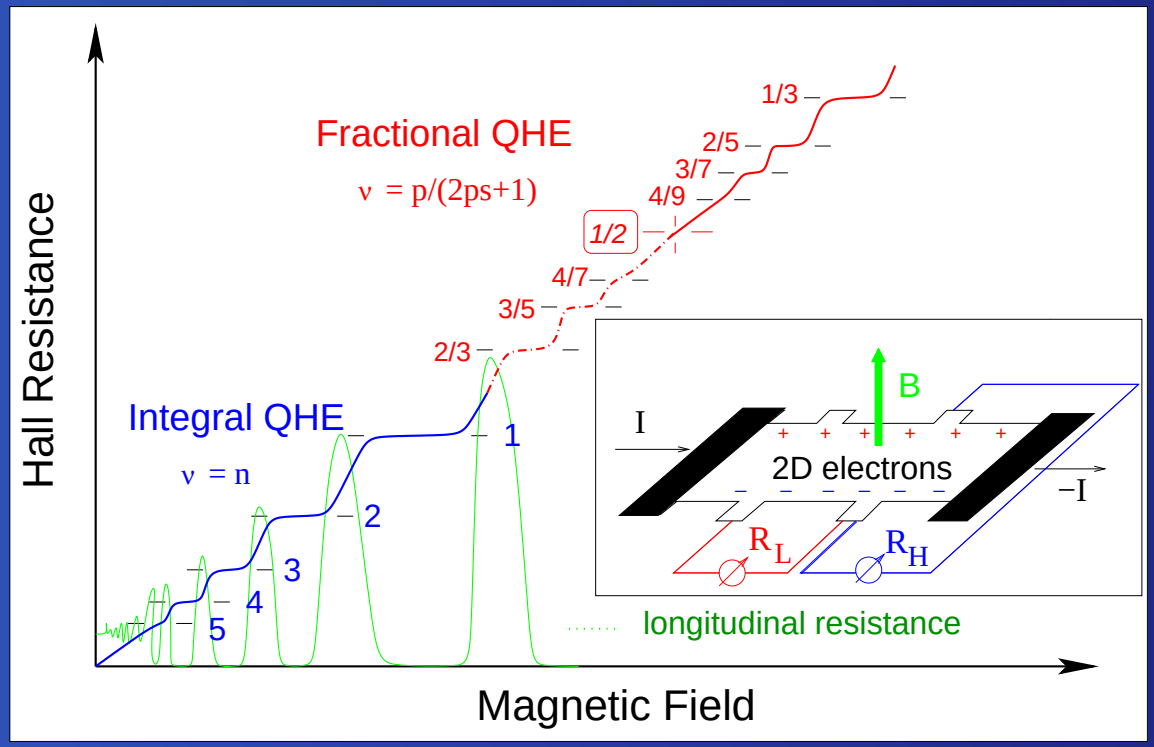

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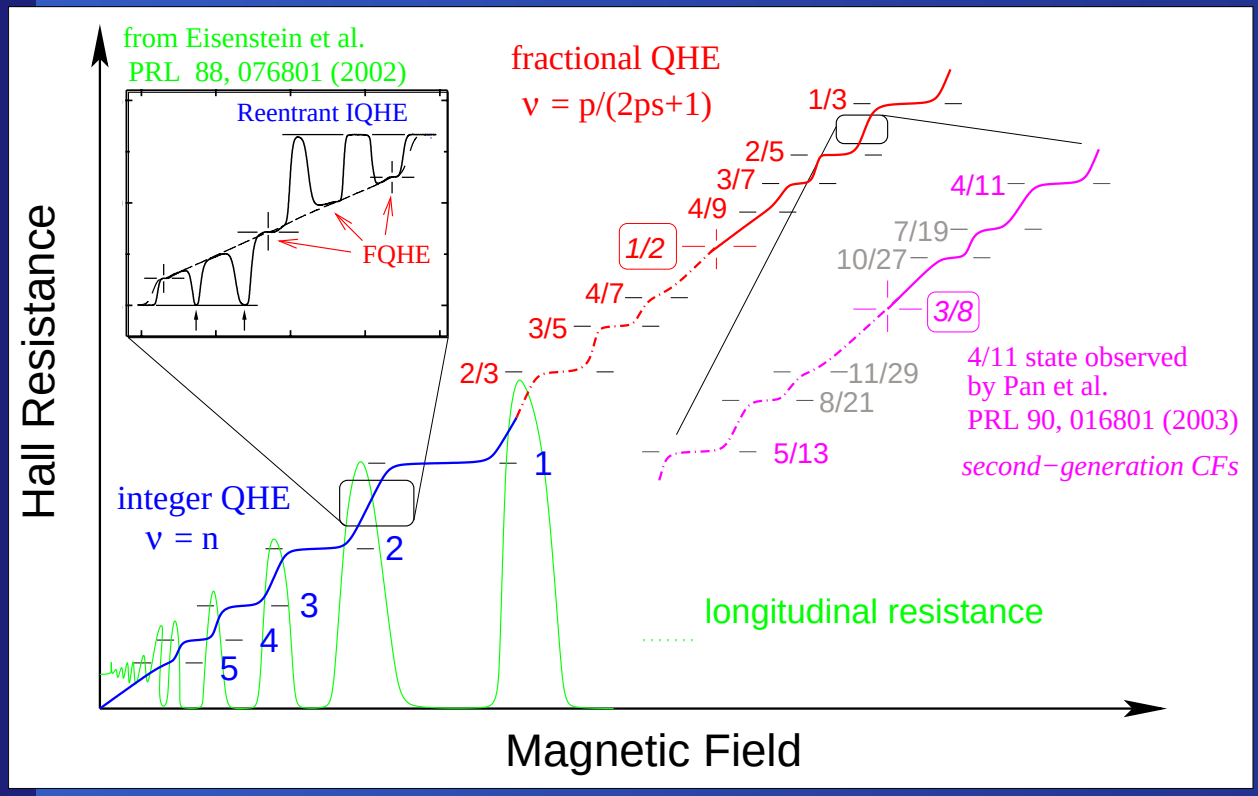

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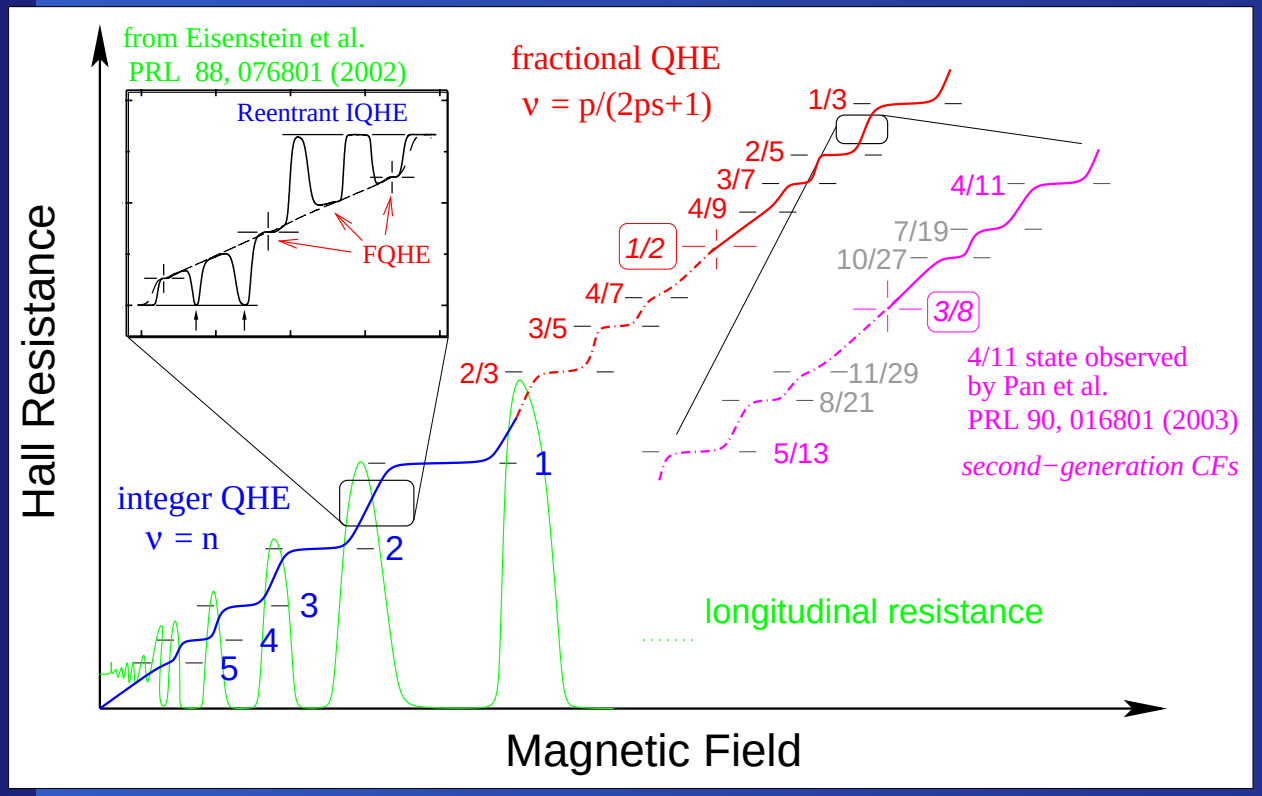

Non-monotonic behavior of the Hall resistance, reentrant IQHE

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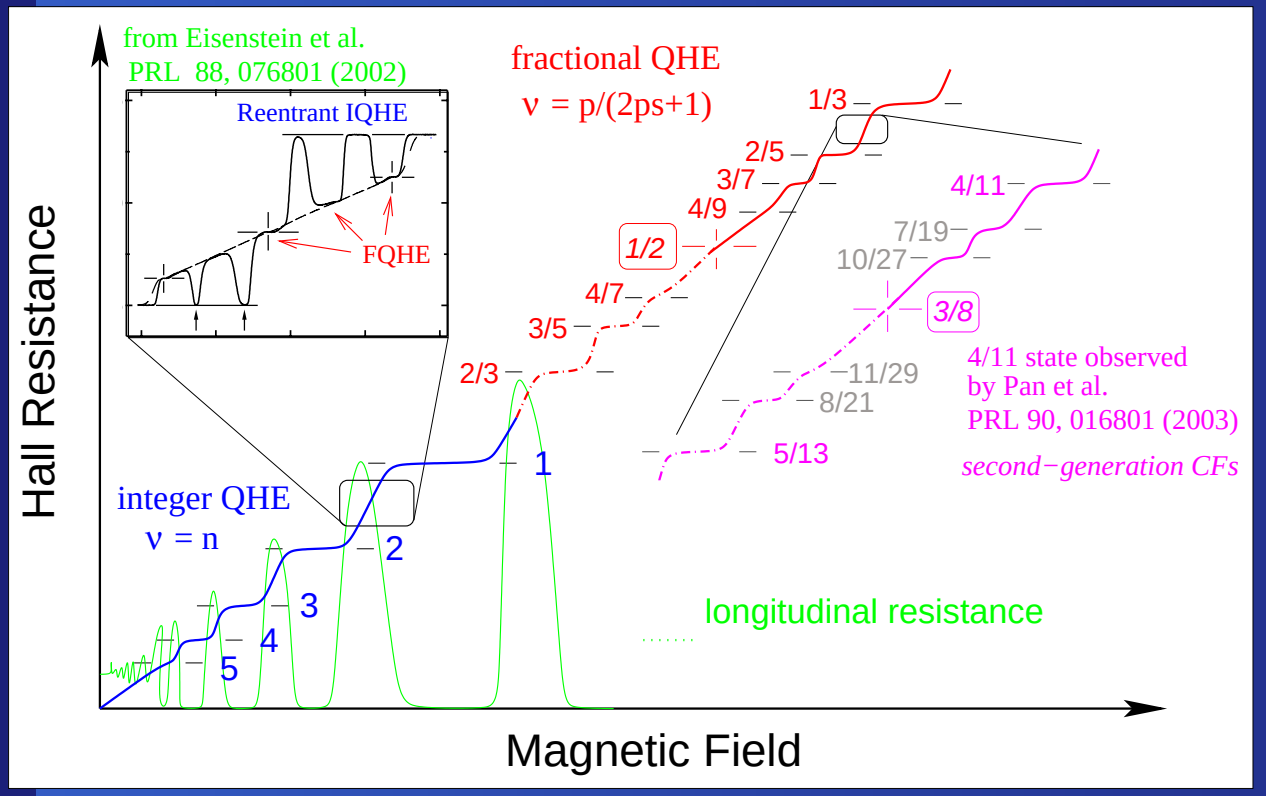

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(Second generation of CFs → see poster in this Conference)

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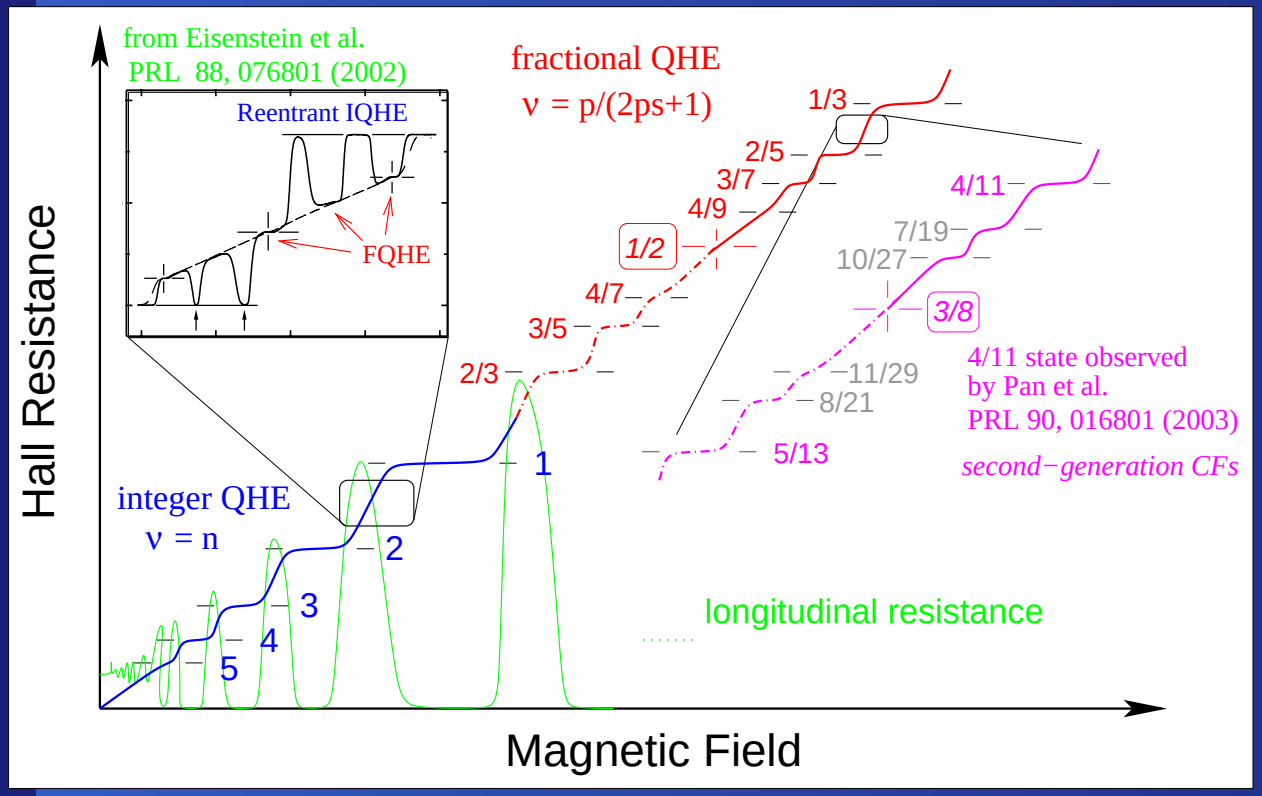

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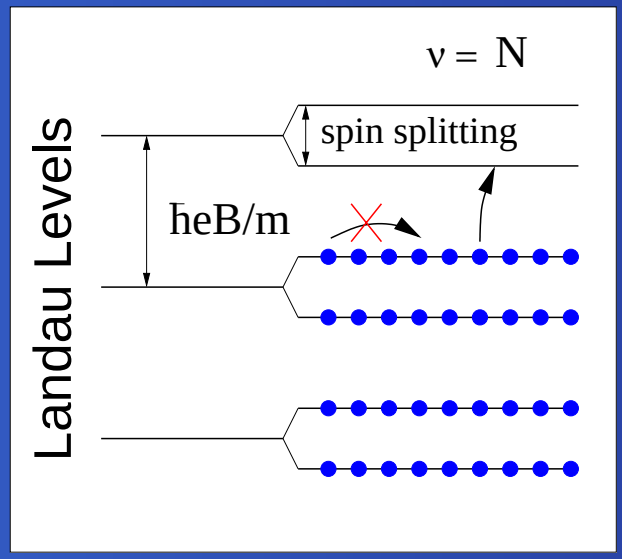
New phases due to strong correlations (Coulomb interaction): competition between electron solids and quantum liquids

Strong Correlations in LLs

- Integral filling



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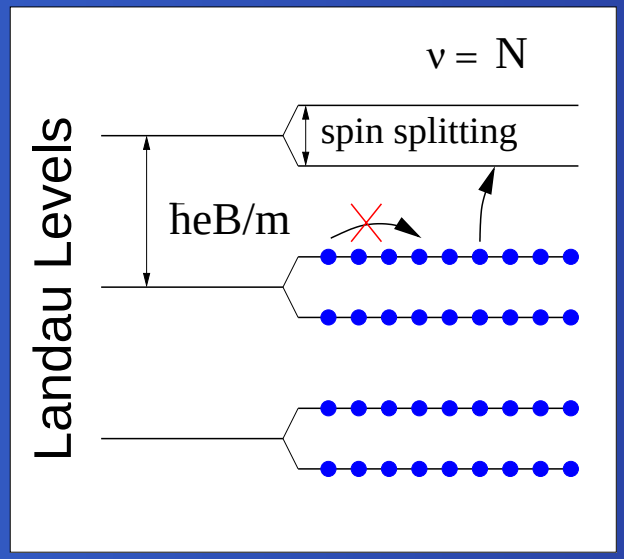

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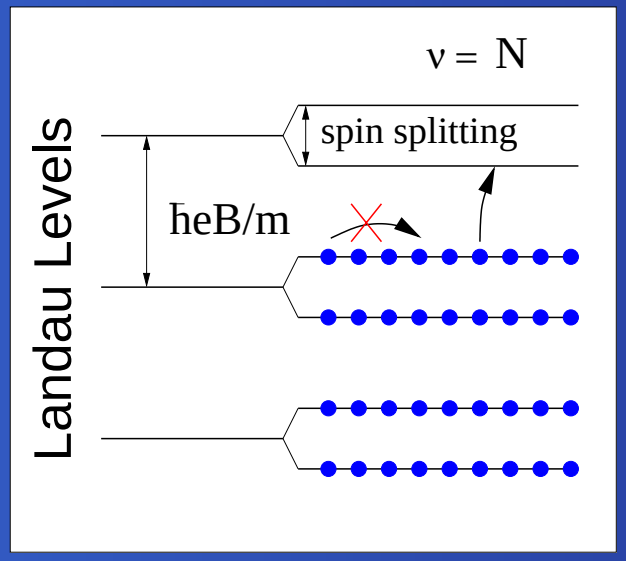

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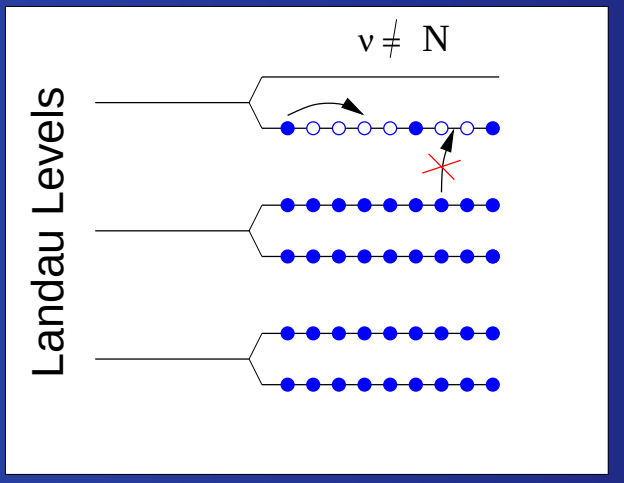

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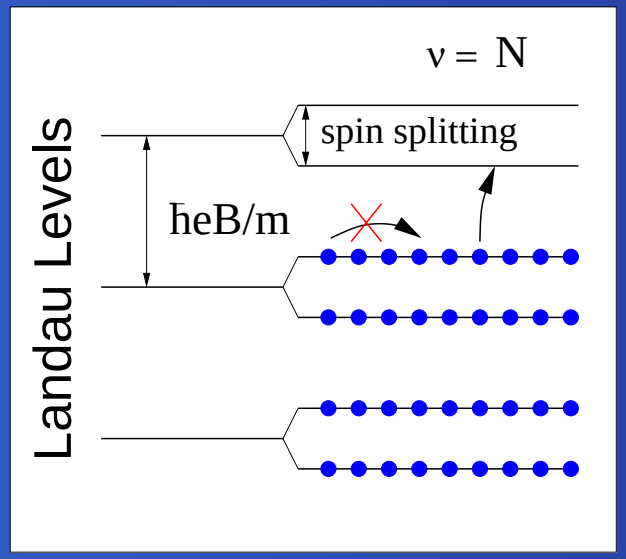

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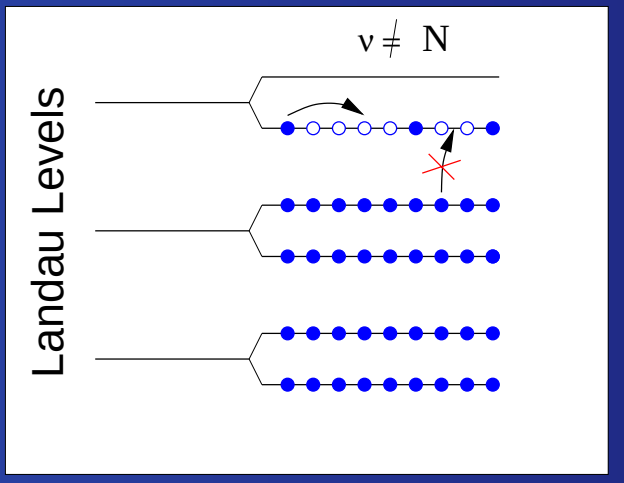

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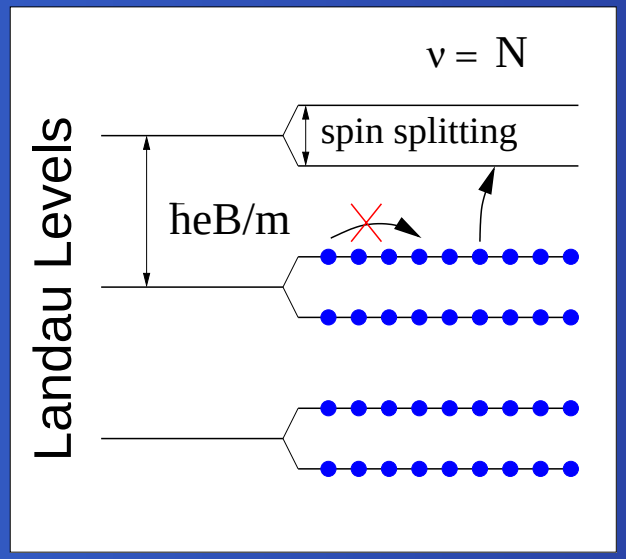

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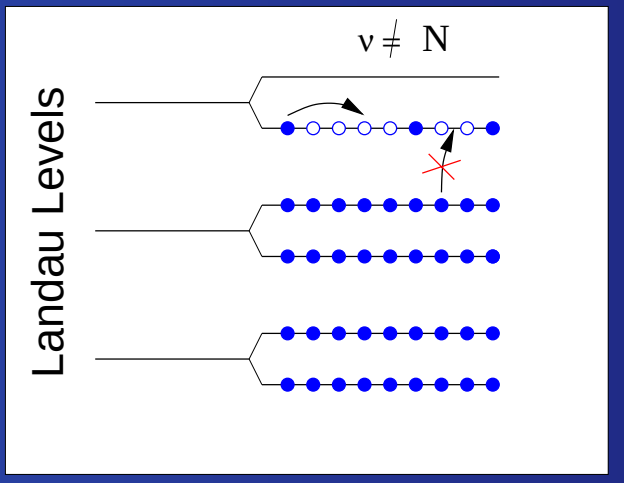

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Coulomb interaction relevant - limit of strong correlations

Effective Model

Hamiltonian of interacting electrons in fixed LL

$$\hat{H} = \frac{1}{2}\sum_{\mathbf{q}} v_n(q)\bar{\rho}(-\mathbf{q})\bar{\rho}(\mathbf{q}), \qquad \text{with} \;\; v_n(q) = \frac{2\pi e^2}{\epsilon q}\left[F_n(q)\right]^2$$

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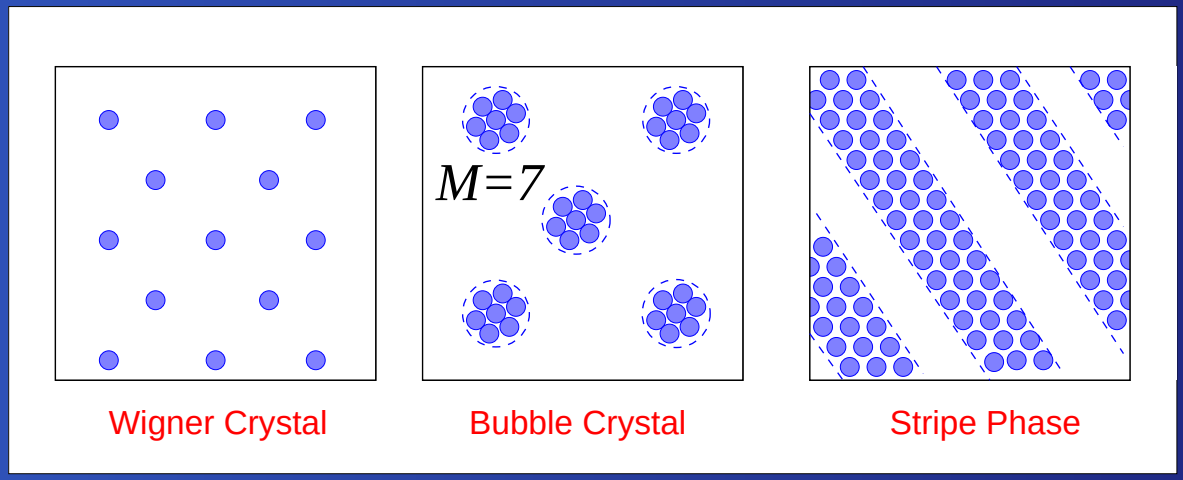
Common model for all LLs!

Electron-Solid Phases (I)



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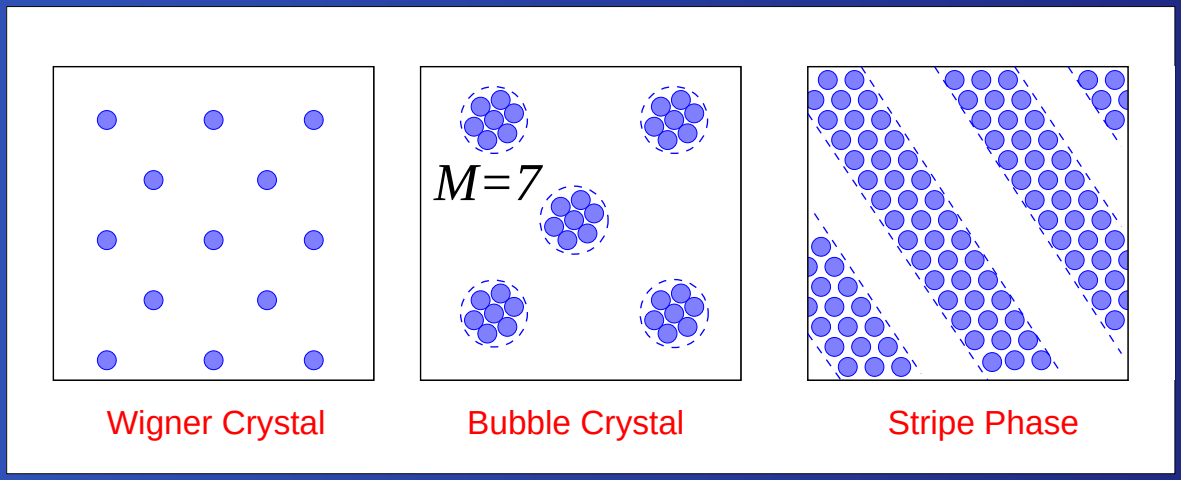

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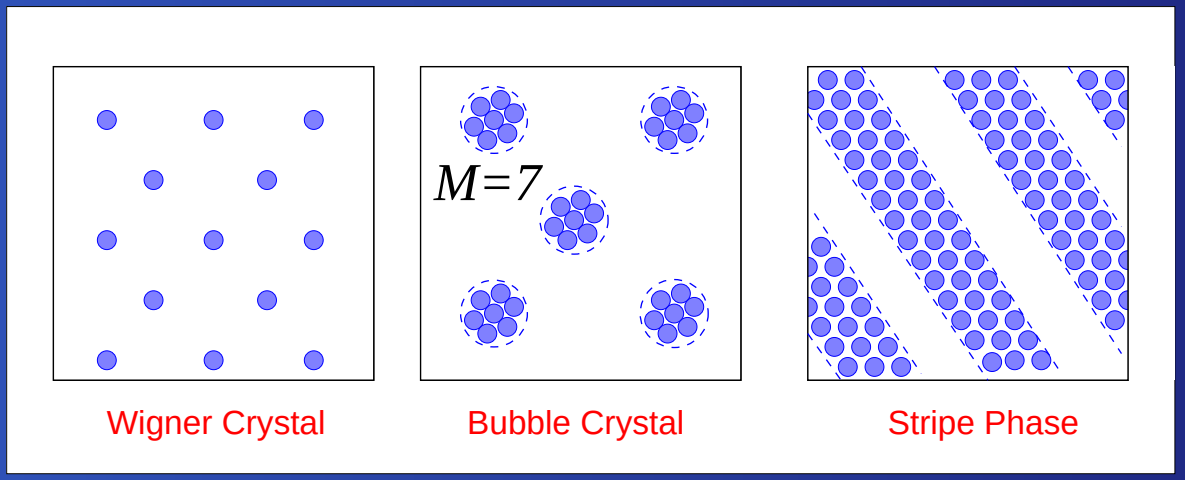

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- $\langle \bar{\rho}(\mathbf{q}) \rangle$ plays role of *order parameter*

Electron-Solid Phases (II)

- Energy of bubble crystal (M electrons per bubble)

$$E^B_{coh}(n; M, \bar{\nu}) = \frac{n_B \bar{\nu}}{M} \sum_l u_n^{HF}(\mathbf{G}_l) \frac{J_1^2(\sqrt{2M}|\mathbf{G}_l|)}{|\mathbf{G}_l|^2}$$

$\mathbf{G}_l$: reciprocal-lattice vectors *($\longrightarrow$ crystal symmetry)*

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- Energy of stripe phase
  ($\Lambda_S \sim R_C$: variational parameter)

$$E^S_{coh}(n; \Lambda_S, \bar{\nu}) = \frac{n_B}{2\pi^2 \bar{\nu}} \sum_{l \neq 0} u_n^{HF}\left(q = \frac{2\pi}{\Lambda_S} l\right) \frac{\sin^2(\pi \bar{\nu} l)}{l^2}$$

Quantum-Liquid Phases

- At $\bar{\nu} = 1/(2s+1)$: Laughlin wavefunctions

$$E_{coh}^{Laughlin}(n;s) = \frac{\bar{\nu}}{\pi} \sum_{m=0}^{\infty} c_{2m+1}^{s} V_{2m+1}^{n}$$

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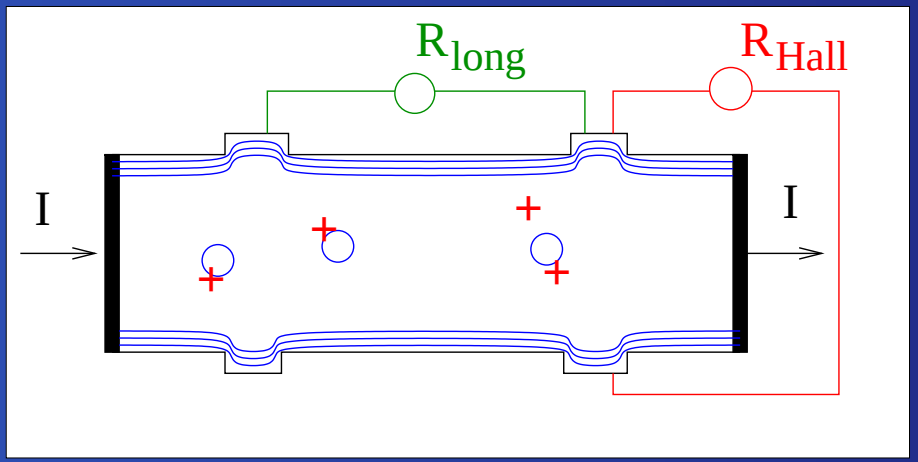
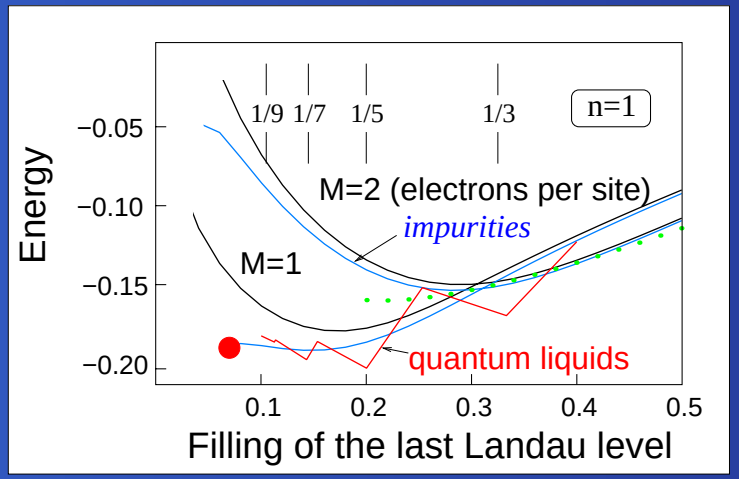
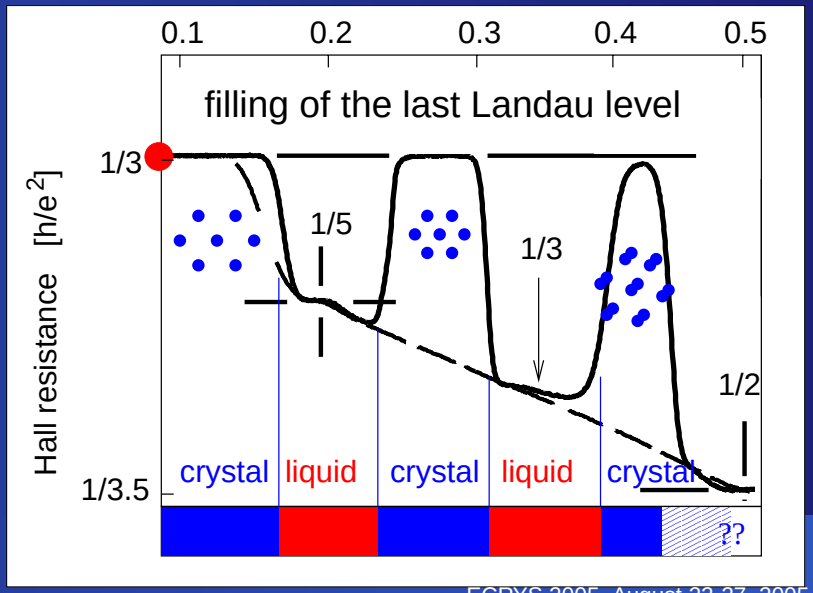
  - $\Delta^{n}(s)$ calculated in *Hamiltonian theory of the FQHE*

[Murthy and Shankar, RMP 75, 1101 (2003)]

Results for $n = 1$







[PRB 68, 241302(R) (2003); PRB 69, 115327 (2004)]

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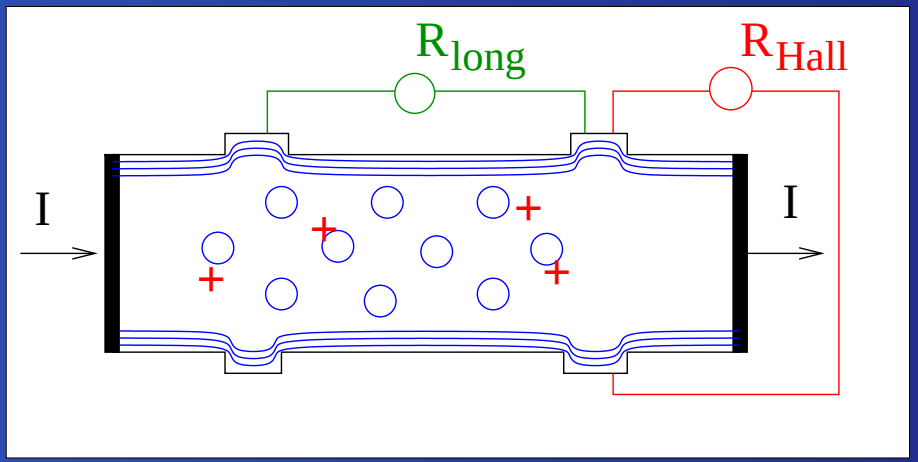

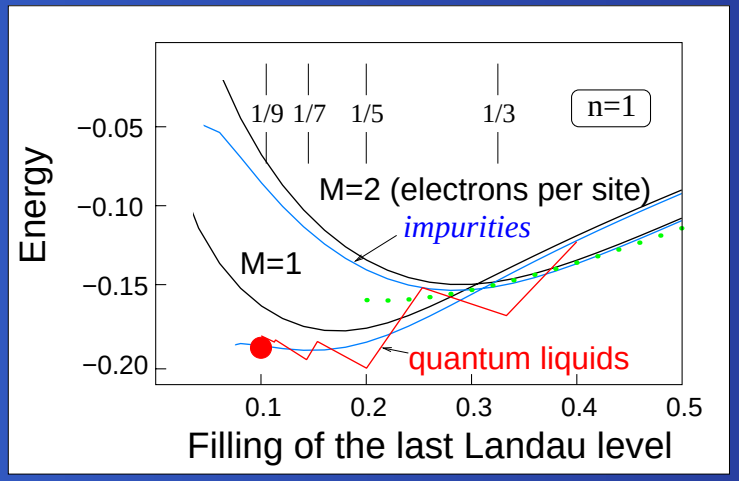

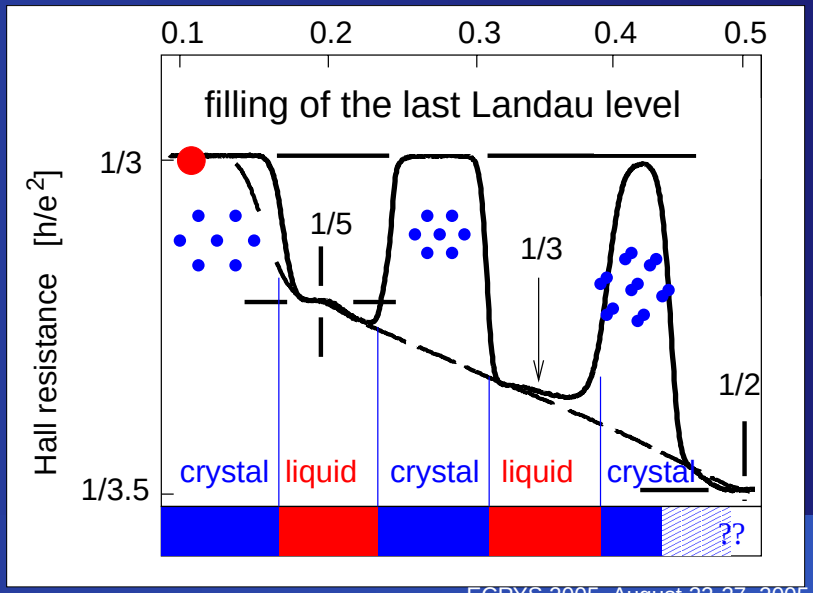

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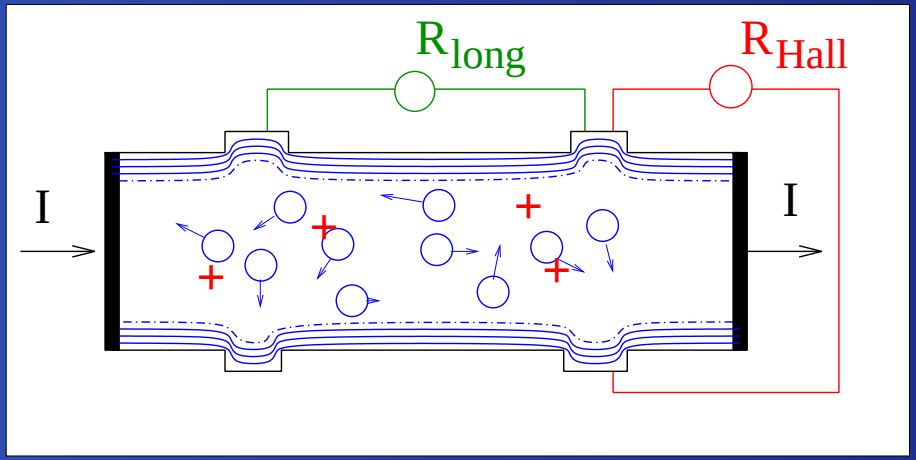

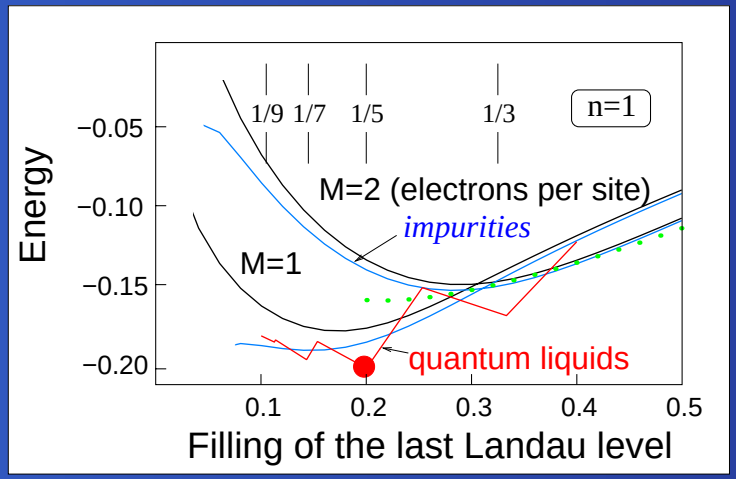

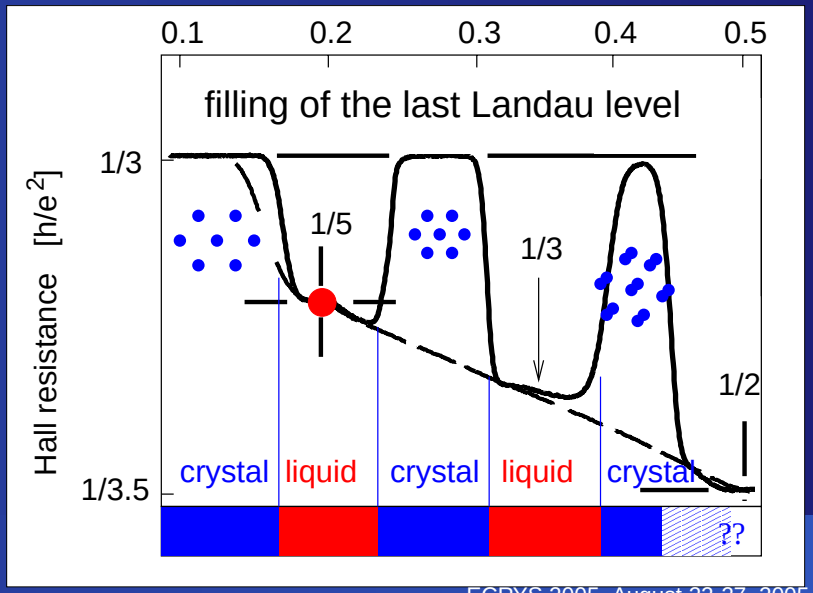

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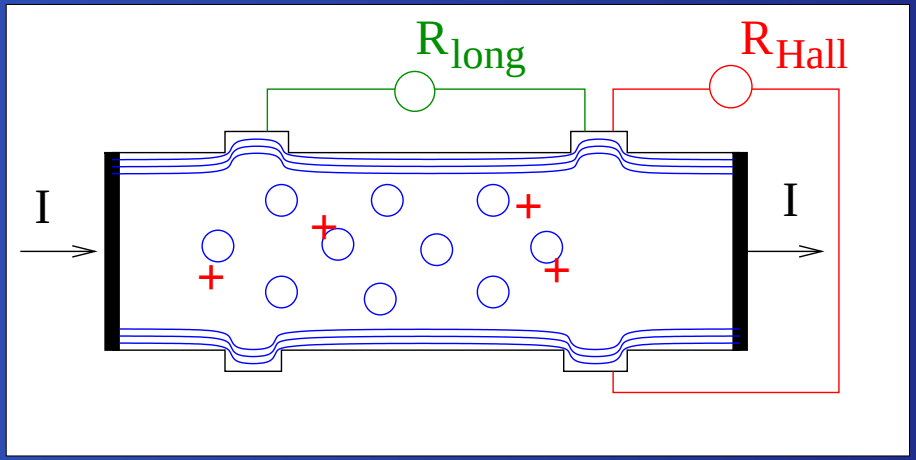

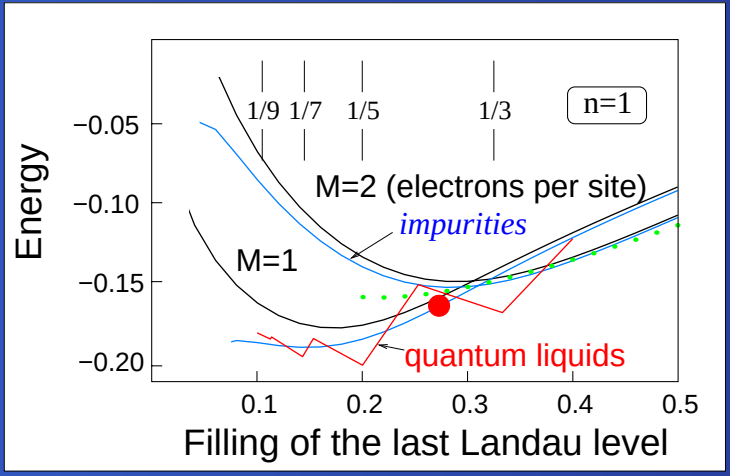

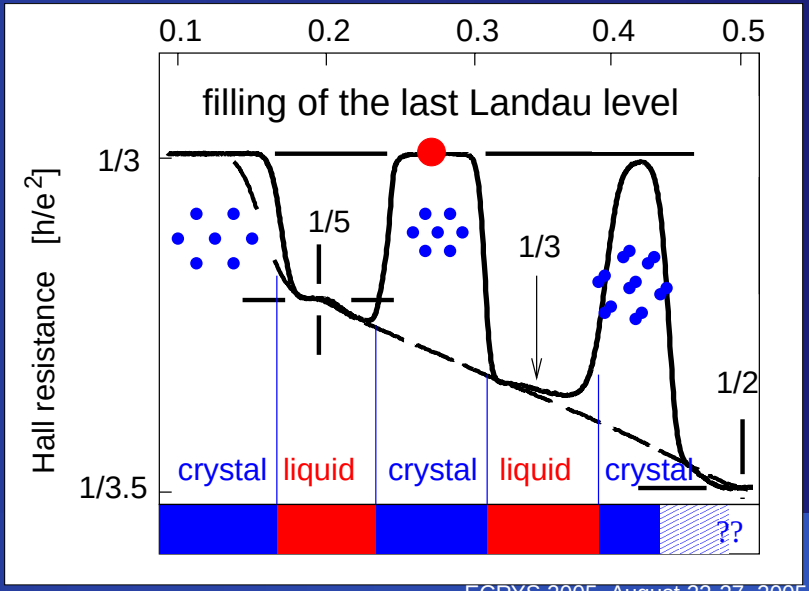

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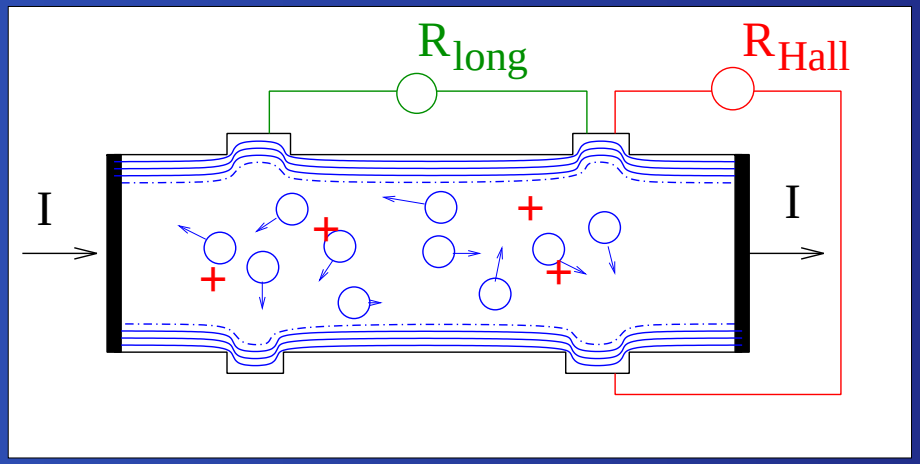

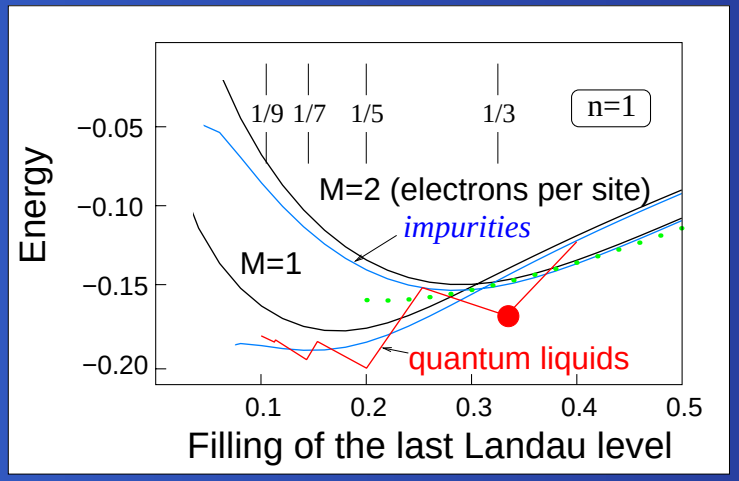

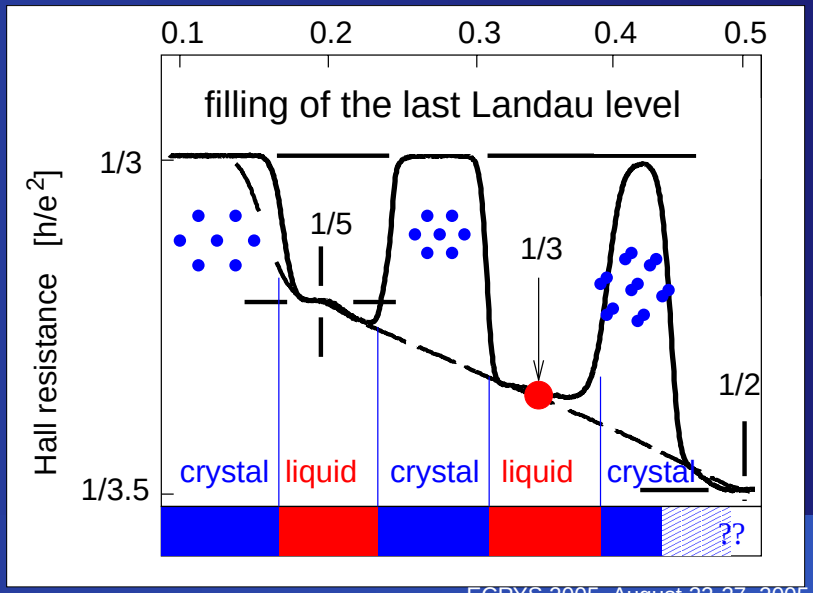

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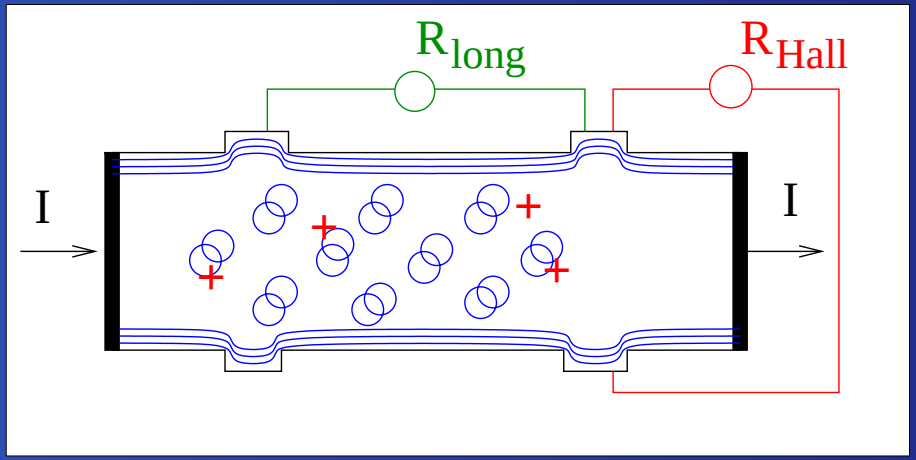

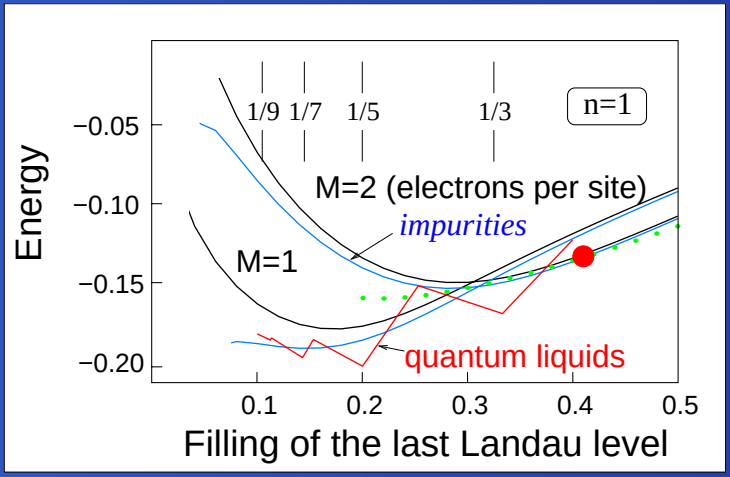

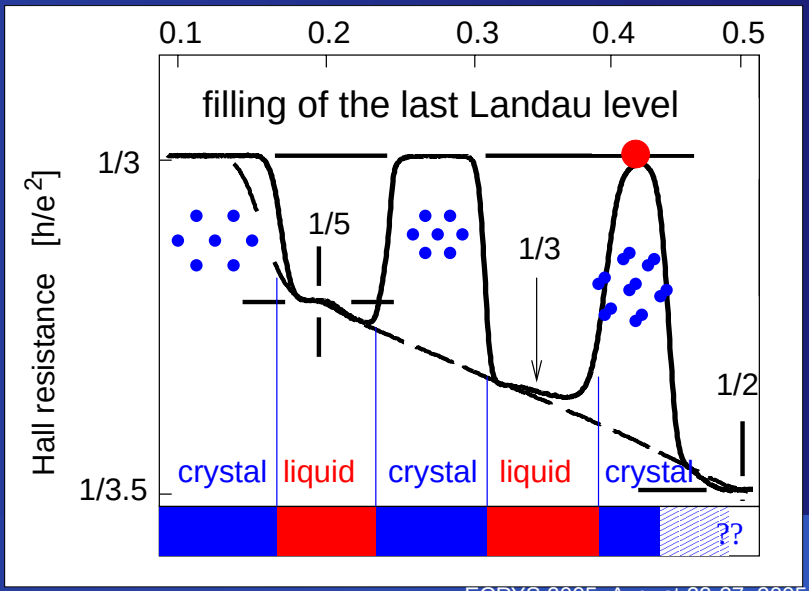

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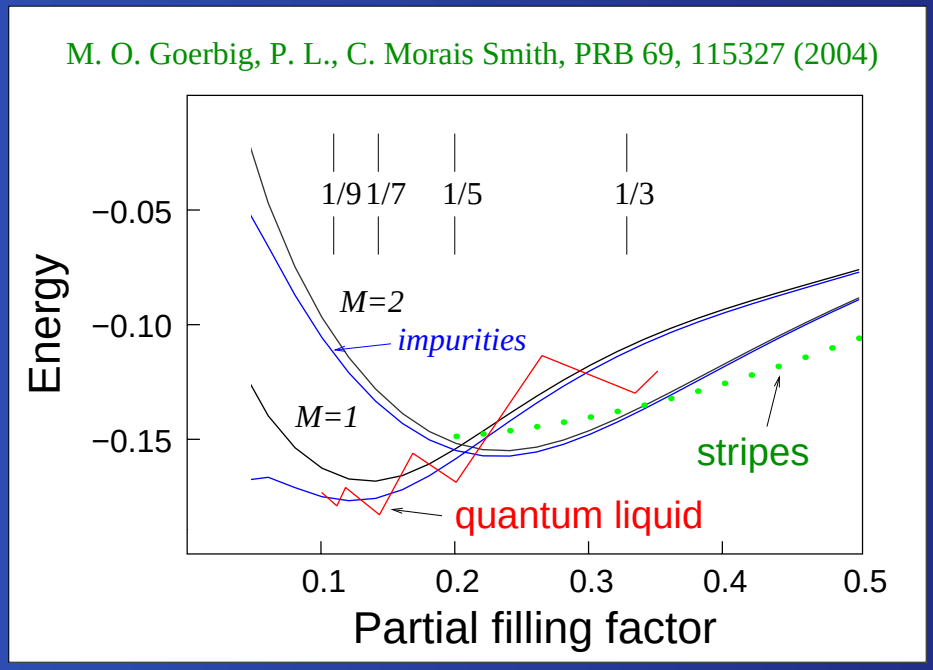
Results ($n = 2$, with impurities)

- Impurities lower energy of electron solids (blue curves)



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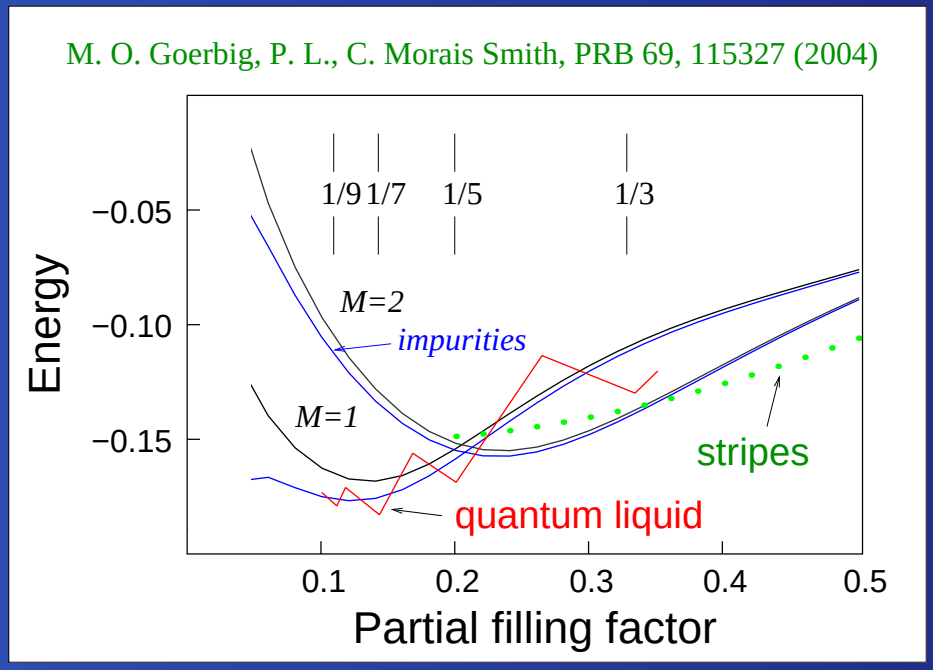

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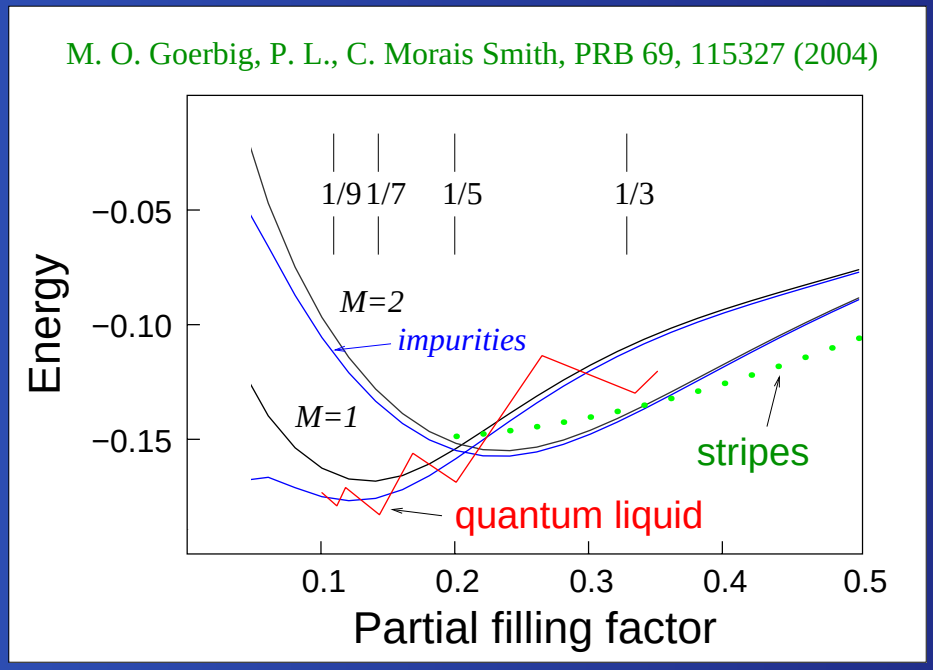

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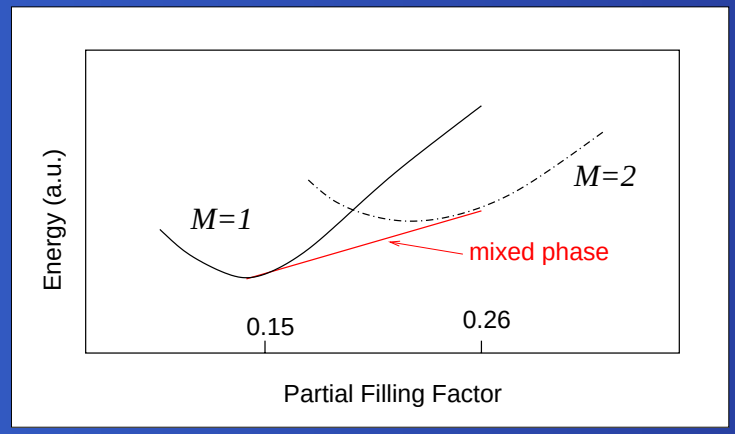

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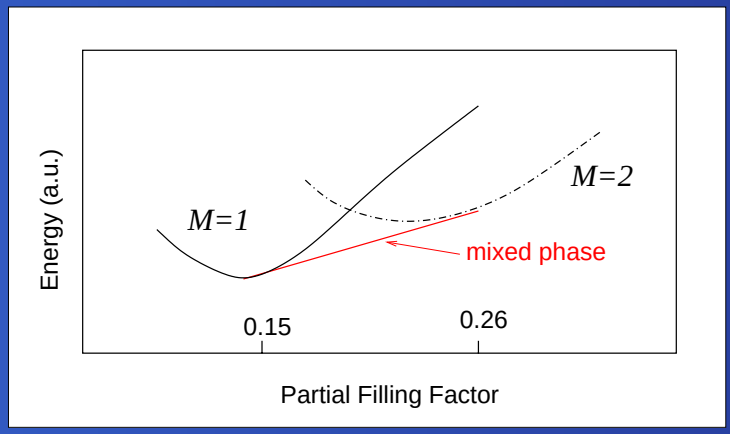

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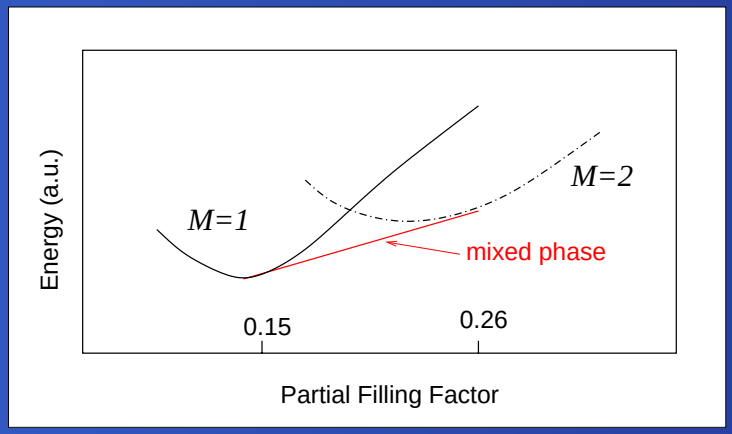

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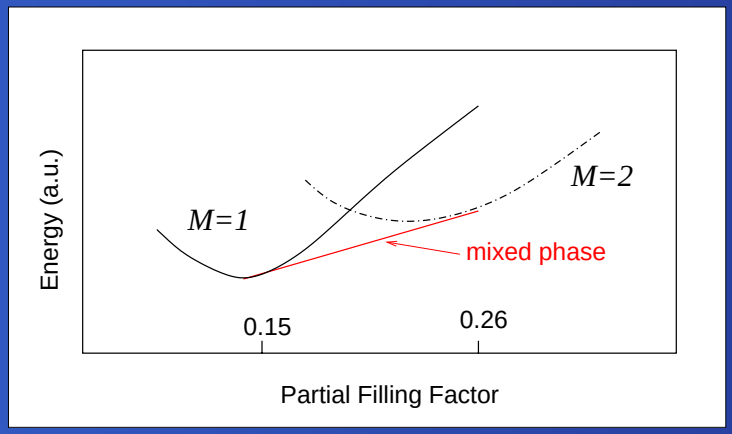

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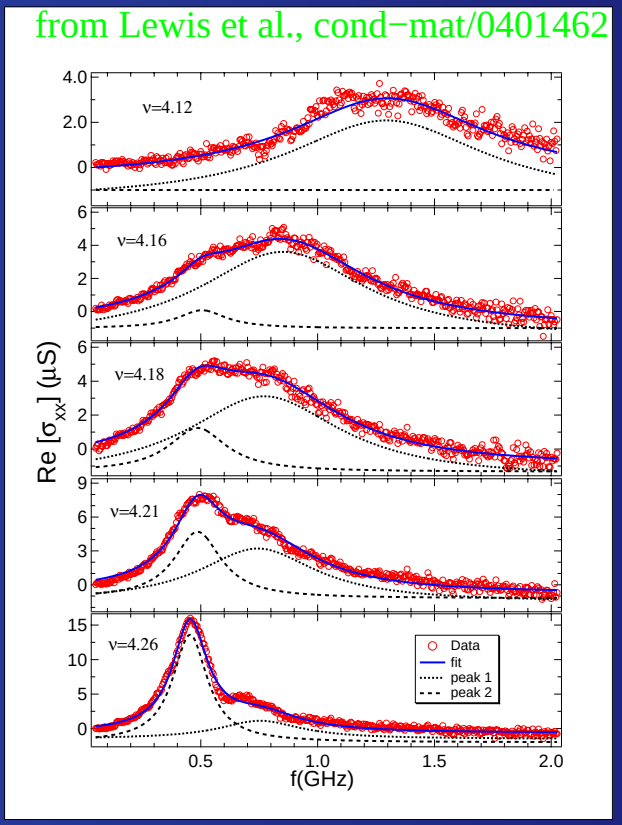

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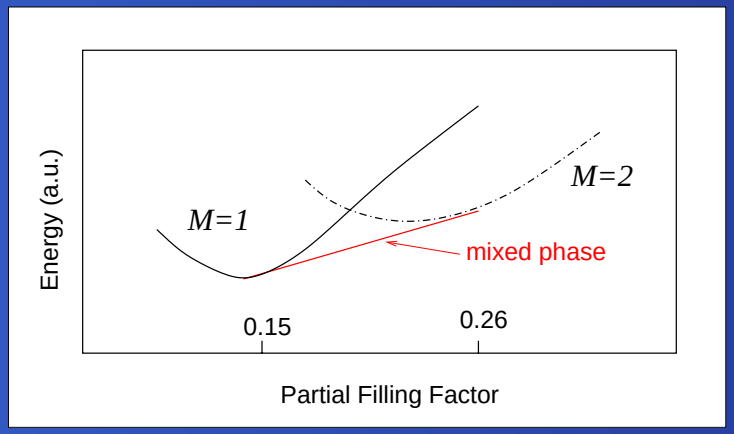

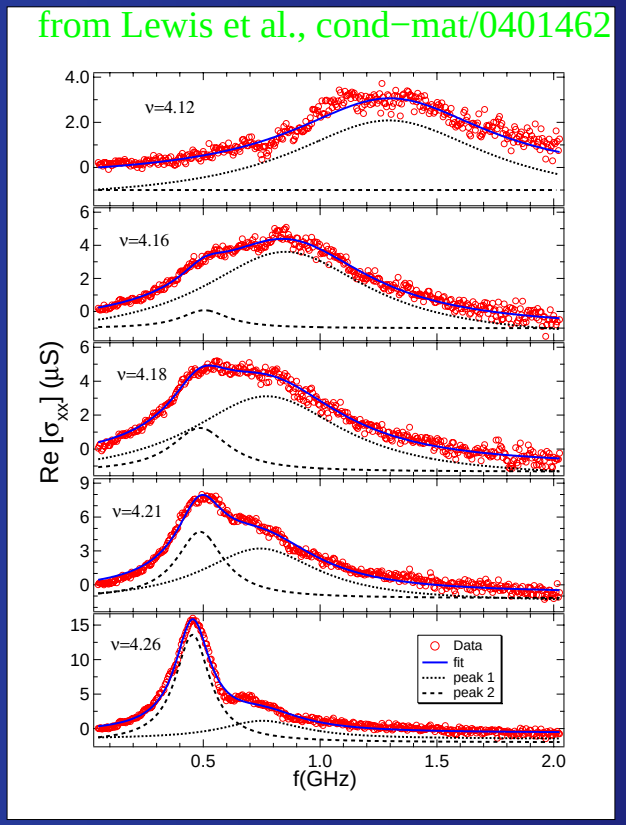

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- Pinning peaks at $\omega_p/\omega_C \sim M^{-1/2}\bar{\nu}^{-3/2}$

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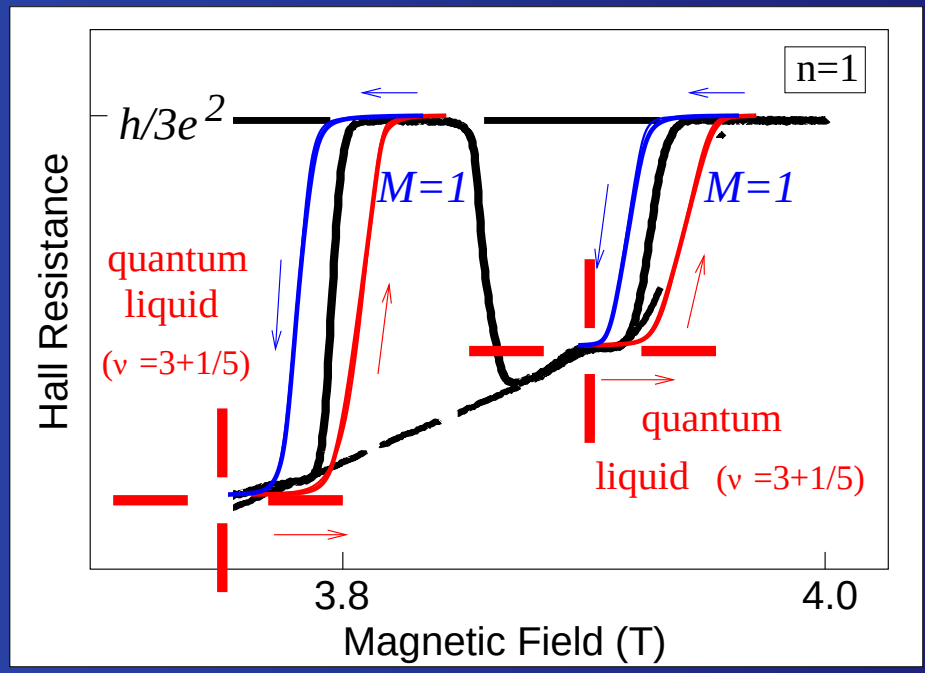

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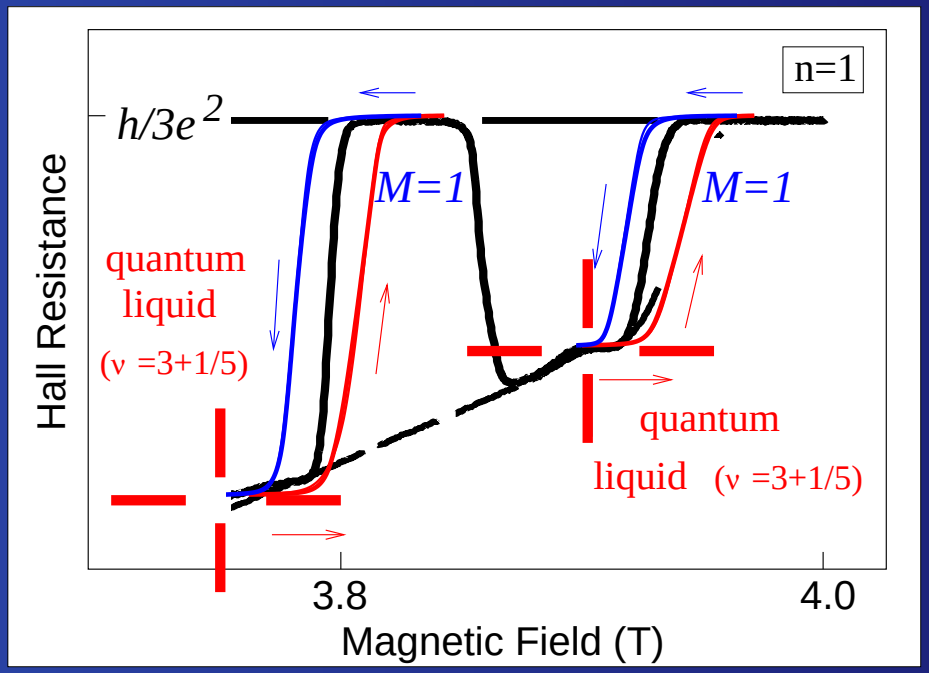
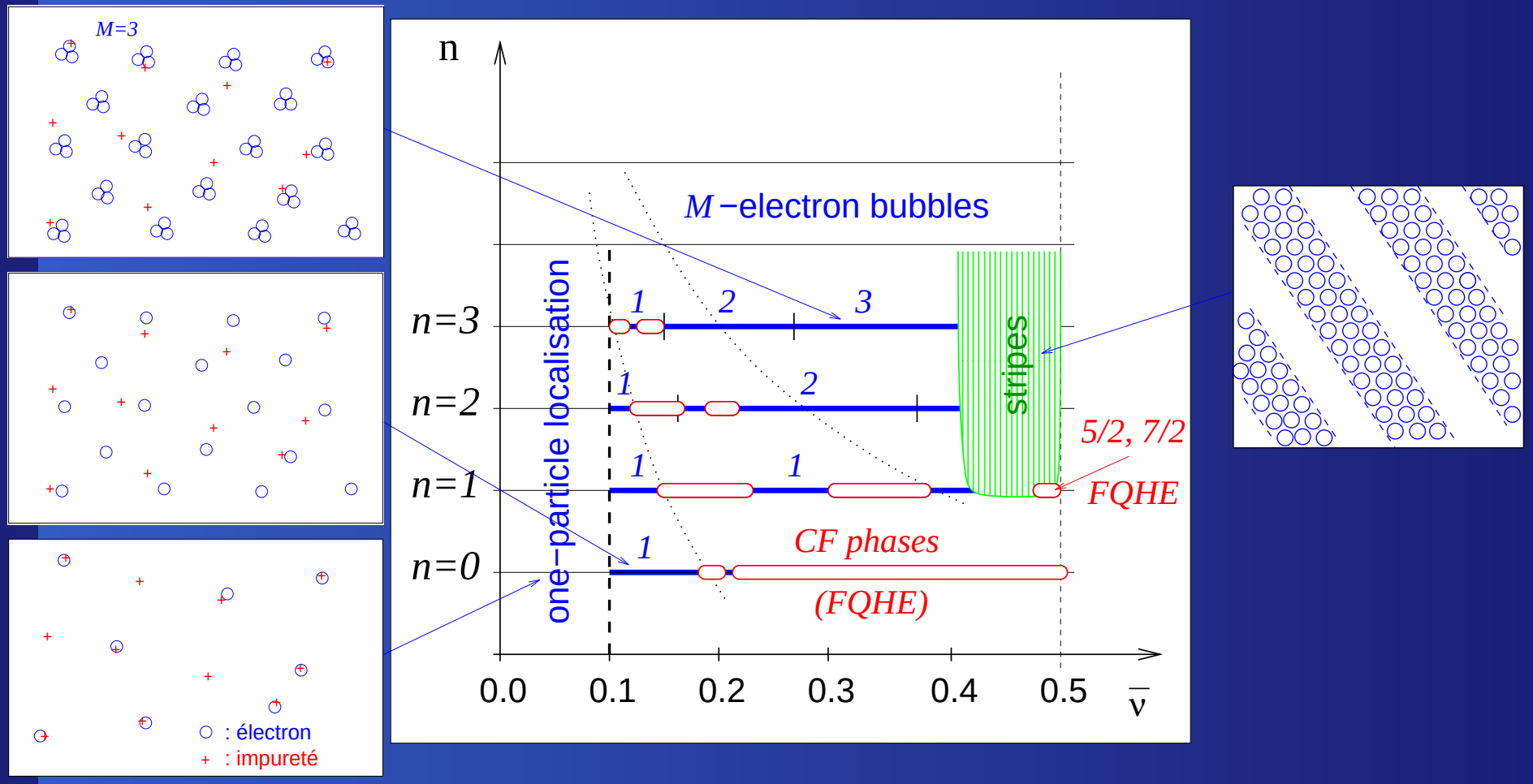


Hysteresis in Hall resistance expected

not observed yet

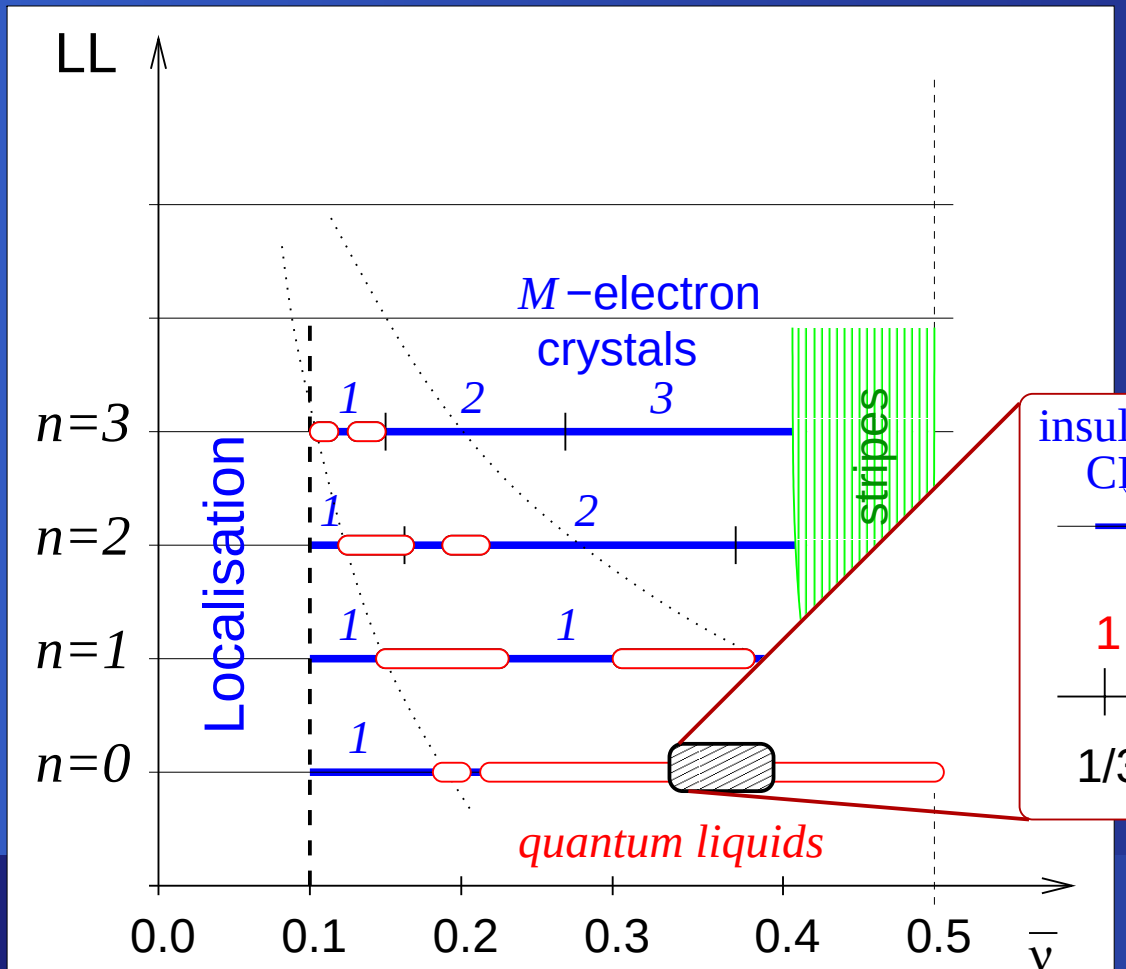
Phase Diagram – Electronic Model



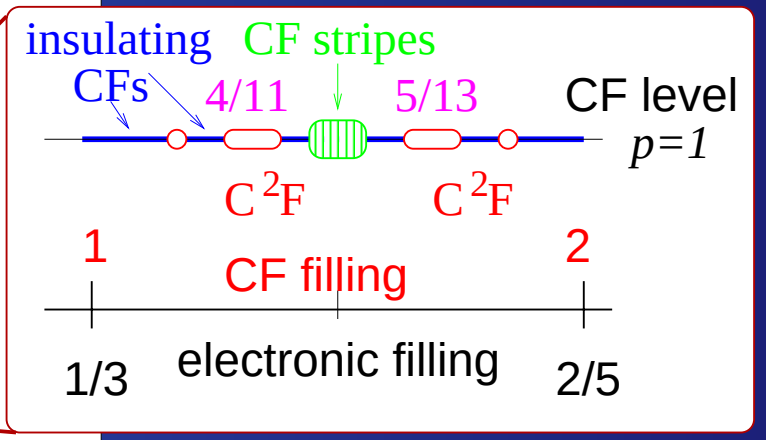
M.O.G., P. Lederer, C. Morais Smith, PRB 69, 115327 (2004)

What about Lowest LL?

Re-entrance of Fractional Quantum Hall Effect? See poster by M. O. Goerbig



Zoom in



CF phases

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